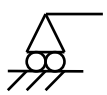
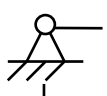


Liaisons

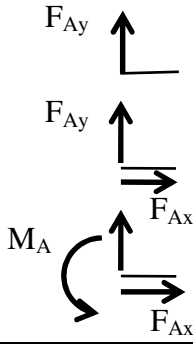
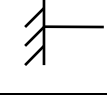
Appuis simple



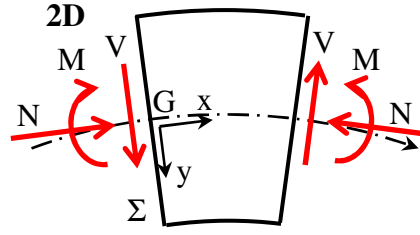
Articulation



Encastrement



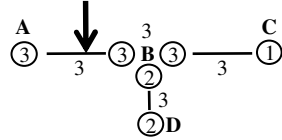
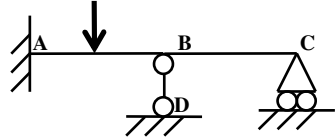
Sollicitations Définition



$$\begin{pmatrix} N \\ V_y \\ V_z \end{pmatrix} = \begin{matrix} \sum \\ \sum_{\text{gauche}} \\ \sum_{\text{droite}} \end{matrix} \overline{\text{Efforts}} \quad \text{3D}$$

$$\begin{pmatrix} T \\ M_y \\ M_z \end{pmatrix} = \begin{matrix} \sum \\ \sum_{\text{gauche}} \\ \sum_{\text{droite}} \end{matrix} \overline{\text{Moments}}_G$$

Hyperstatisme



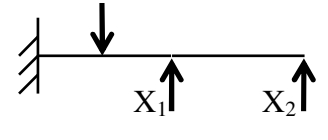
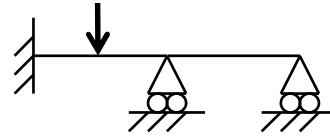
Barres : $b = 3$

Equations d'équilibre noeud : $\sum N_i = 3 \quad j \text{ Equations}$

Inconnues de liaison : $\sum l_i = 14 \quad \textcircled{1} \text{ Inconnues}$

$$\sum l_i - (3b + \sum N_i) = 2 \quad \text{Hyperstatisme degré 2}$$

Structure isostatique associée



Hyperstatisme degré 2
-2 degré de liberté

Structure isostatique associée

Caractéristiques géométriques

Aire

$$A = \int_{\Sigma} dS$$

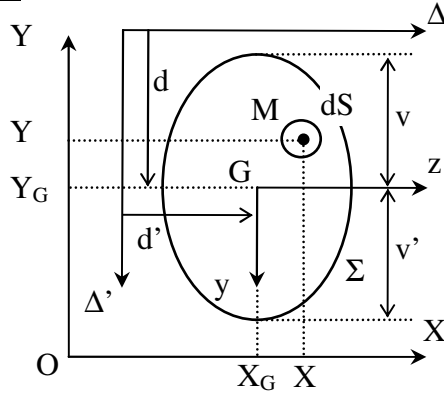
Moment statique

$$S_x = \int_{\Sigma} Y dS \quad \text{et} \quad S_y = \int_{\Sigma} X dS$$

Centre de Gravité

$$S_y = S_z = 0$$

$$Y_G = \frac{S_x}{A} \quad \text{et} \quad X_G = \frac{S_y}{A}$$



Moment d'inertie

$$I_x = \int_{\Sigma} Y^2 dS \quad \text{et} \quad I_y = \int_{\Sigma} X^2 dS$$

Produit d'inertie

$$I_{xy} = \int_{\Sigma} XY dS$$

Inerties principales (min et max par G)

$$I_z = \int_{\Sigma} y^2 dS \quad \text{et} \quad I_y = \int_{\Sigma} z^2 dS$$

$$I_{yz} = \int_{\Sigma} yz dS = 0$$

Les axes des inerties principales définissent le repère local : (G, y, z)

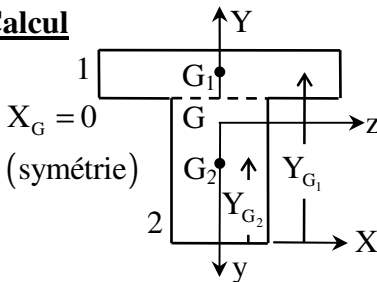
Rayon de giration :

$$i_z = \sqrt{\frac{I_z}{A}} \quad \text{et} \quad i_y = \sqrt{\frac{I_y}{A}}$$

Théorème d'HUYGENS

$$I_{\Delta} = I_z + Ad^2, \quad I_{\Delta'} = I_y + Ad'^2, \quad I_{\Delta\Delta'} = I_{yz} + Add'$$

Calcul

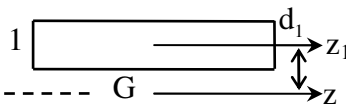


$X_G = 0$
(symétrie)

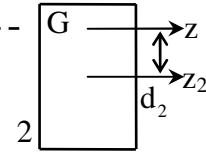
$$S_x(1) = A(1) Y_{G1}$$

$$S_x(2) = A(2) Y_{G2}$$

$$Y_G = (S_x(1) + S_x(2)) / (A(1) + A(2))$$



$$I_z(1) = I_{z1}(1) + A(1)d_1^2$$

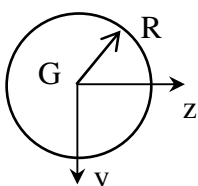


$$I_z(2) = I_{z2}(2) + A(2)d_2^2$$

$$I_z = I_z(1) + I_z(2)$$

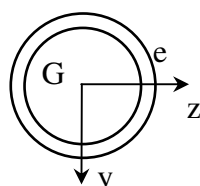
$$I_y = I_y(1) + I_y(2)$$

Exemples



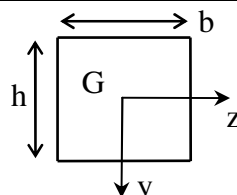
$$I_z = \frac{\pi R^4}{4}$$

$$I_y = \frac{\pi R^4}{4}$$



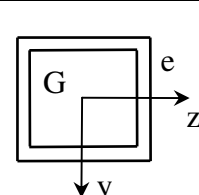
$$I_z = \pi e R^3$$

$$I_y = \pi e R^3$$



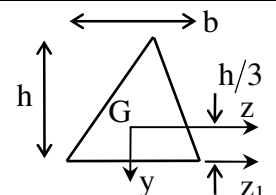
$$I_z = \frac{bh^3}{12}$$

$$I_y = \frac{hb^3}{12}$$



$$I_z = \frac{eh^2}{6} (3b + h)$$

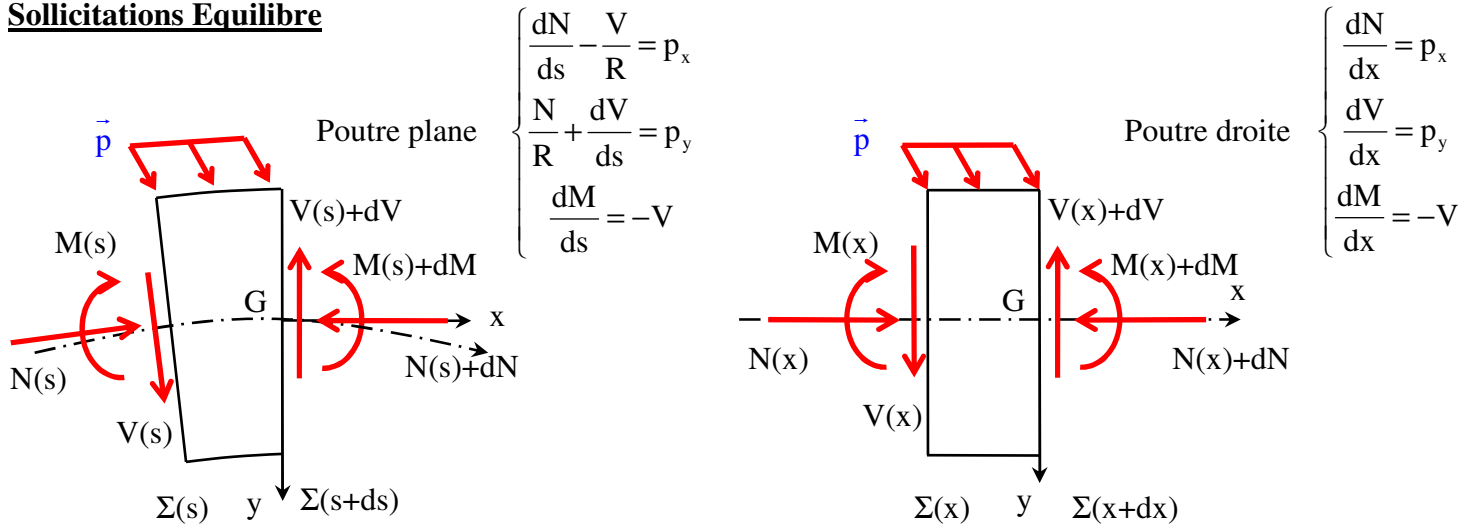
$$I_y = \frac{eb^2}{6} (3h + b)$$



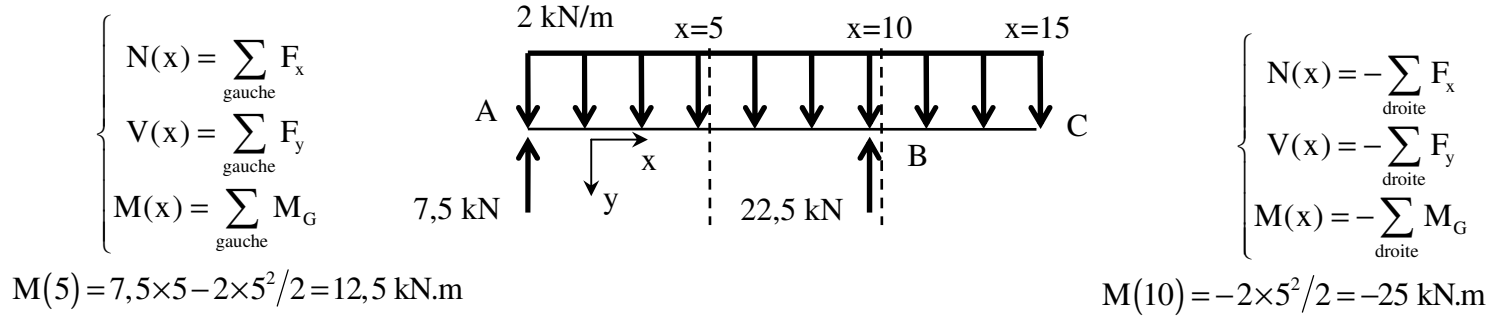
$$I_z = \frac{bh^3}{36}$$

$$I_{z1} = \frac{bh^3}{12}$$

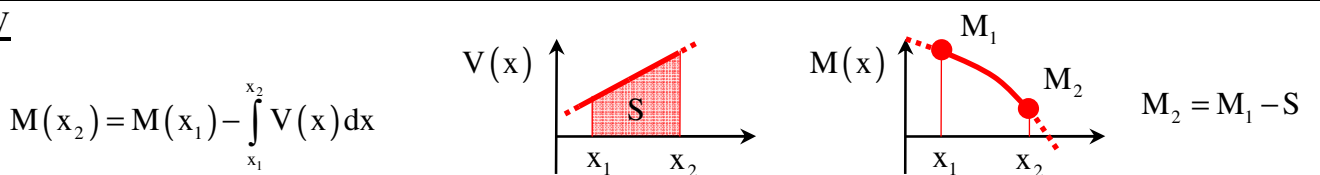
Sollicitations Equilibre



Sollicitations Expression 2D



Relation M-V



Sollicitations Tracé

Effets des actions ponctuelles

Effets des actions réparties

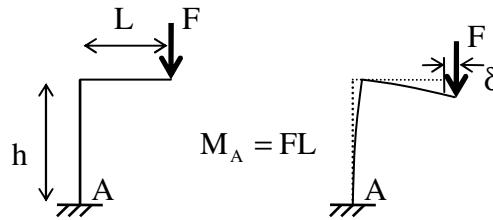
Action de liaison	Force suivant y	Couple suivant z	Action répartie	Pas d'action répartie
	Saut de F	Sans effet	Droite Pente p	Droite horizontale
	Changement de pente	Saut de C	Arc de parabole	Droite inclinée

Hypothèses de la MMC

Hypothèses des Petites Perturbations

Petites déformations

$$\varepsilon(M) = \varepsilon_{ij} \text{ avec } \varepsilon_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$$



Petits déplacements

Equations d'équilibre dans la configuration non déformée

Loi de Comportement Elastique Linéaire Homogène Isotrope : Loi de HOOKE

$$\left. \begin{aligned} \varepsilon &= \frac{1+\nu}{E} \sigma - \frac{\nu}{E} (\text{tr} \sigma) \delta \\ \sigma &= 2\mu \varepsilon + \lambda (\text{tr} \varepsilon) \delta \end{aligned} \right\} \xrightarrow{\text{3D}} \left\{ \begin{aligned} \begin{pmatrix} \varepsilon & \gamma_y/2 & \gamma_z/2 \\ \gamma_y/2 & \varepsilon_T & 0 \\ \gamma_z/2 & 0 & \varepsilon_T \end{pmatrix} & \begin{pmatrix} \sigma = E\varepsilon \\ \tau_y = G\gamma_y \\ \tau_z = G\gamma_z \end{pmatrix} \\ -\frac{\varepsilon_T}{\varepsilon} = \nu & \quad G = \frac{E}{2(1+\nu)} \end{aligned} \right\} \xrightarrow{\text{2D}} \left\{ \begin{aligned} \begin{pmatrix} \sigma & \tau \\ \tau & 0 \end{pmatrix} & \begin{pmatrix} \sigma = E\varepsilon \\ \tau = G\gamma \end{pmatrix} \\ \begin{pmatrix} \varepsilon & \gamma/2 & 0 \\ \gamma/2 & \varepsilon_T & 0 \\ 0 & 0 & \varepsilon_T \end{pmatrix} & \end{aligned} \right\}$$

Hypothèses de la RDM

Principe de NAVIER BERNOULLI

$$\varepsilon(y, z) = a + by + cz$$

Principe de SAINT VENANT

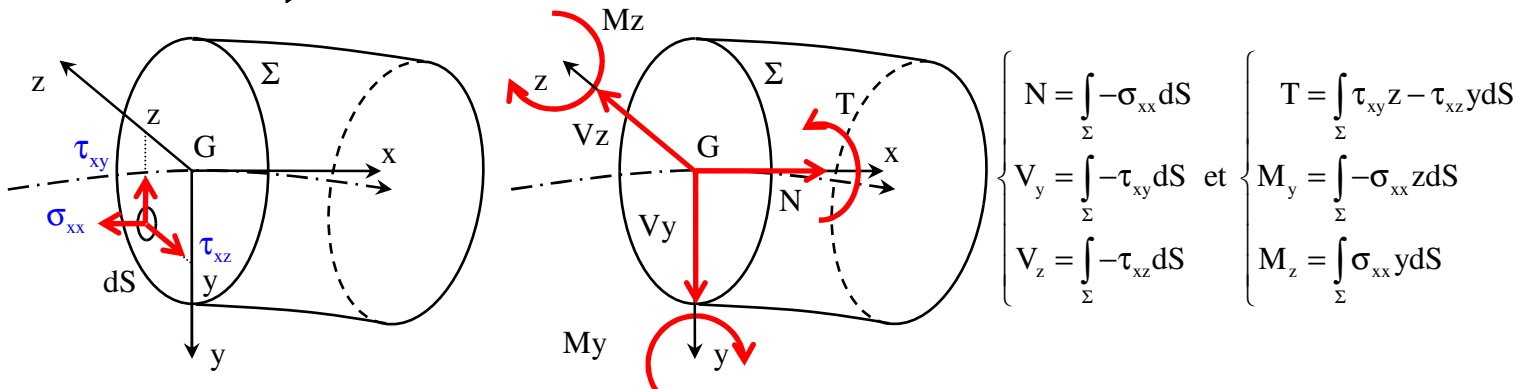
$$\sigma(N, M_y, M_z) \\ \tau_y, \tau_z(V_y, V_z, T)$$

HOOKE

3D $\sigma(y, z) = -\frac{N}{A} + \frac{M_z}{I_z} y - \frac{M_y}{I_y} z$

2D $\sigma(y, z) = -\frac{N}{A} + \frac{M}{I_z} y$

Contraintes Sollicitations



Sollicitations Contraintes normales 2D

N - M=0 Traction-Compression Simple

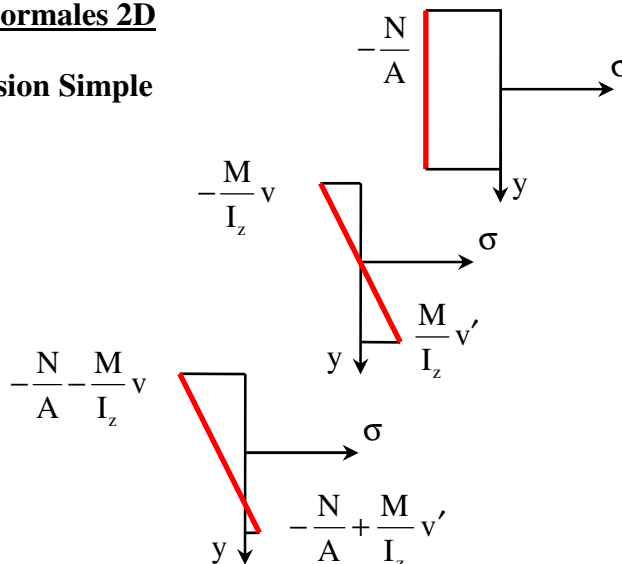
$$\sigma(y) = -\frac{N}{A}$$

N=0 - M Flexion Simple

$$\sigma(y) = \frac{M}{I_z} y$$

N - M Flexion Composée

$$\sigma(y) = -\frac{N}{A} + \frac{M}{I_z} y$$



Centre de Pression : $C \begin{pmatrix} -\frac{M}{N} \\ 0 \end{pmatrix}$

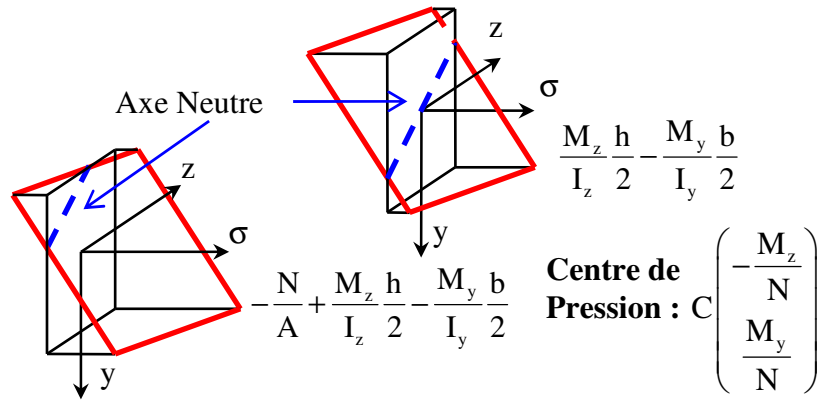
Sollicitations \implies **Contraintes normales 3D**

$N=0 - M_y - M_z$ Flexion déviée

$$\sigma(y, z) = \frac{M_z}{I_z} y - \frac{M_y}{I_y} z$$

$N - M_y - M_z$ Flexion Composée

$$\sigma(y, z) = -\frac{N}{A} + \frac{M_z}{I_z} y - \frac{M_y}{I_y} z$$



Axe neutre

$$\sigma(y, z) = 0 \implies 1 + \frac{y_c}{i_z^2} y + \frac{z_c}{i_y^2} z = 0$$

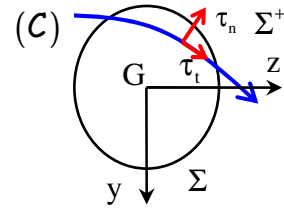
Novau central

Lieux de C tel que section entièrement comprimée

\implies Limite = Axe Neutre tangent à la section

Contraintes tangentes – Théorème de la coupure

$$\int_{(C)} \tau_n dl = -\frac{V_y}{I_z} S_z(\Sigma^+) - \frac{V_z}{I_y} S_y(\Sigma^+)$$



Effort tranchant \implies **Contraintes tangentes**

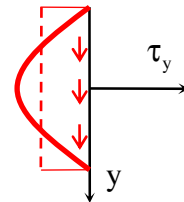
Section massive

$$\tau_y(y) = \frac{V_y}{I_z} \frac{S_z(\Sigma^+)}{b(y)}$$

Section rectangulaire

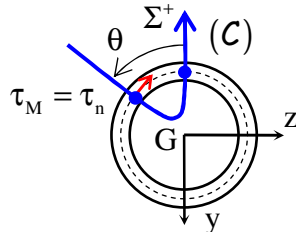
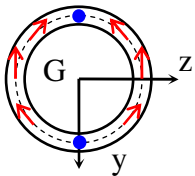
$$\tau_y(y) = -\frac{3}{2} \frac{V_y}{A} \left(1 - \frac{4y^2}{h^2}\right)$$

$$\tau_{y \max} = \frac{3}{2} \frac{V_y}{A}$$



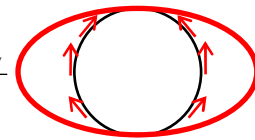
Profil mince fermé symétrique

Flux de cisaillement τe continu



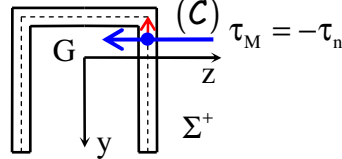
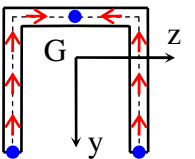
$$\tau_M = \frac{V_y}{\pi Re} \sin \theta = \frac{V_y}{A} 2 \sin \theta$$

$$\tau_{\max} = 2 \frac{V_y}{A}$$



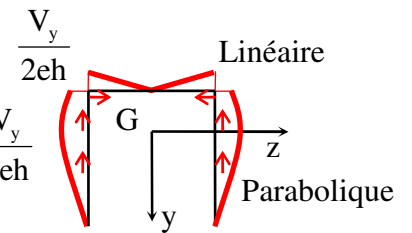
Profil mince ouvert symétrique

Flux de cisaillement τe continu



$$\tau_M = -\tau_n$$

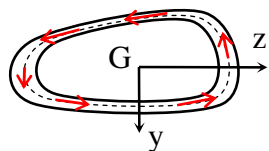
$$\tau_{\max} = \frac{4}{3} \frac{V_y}{2eh}$$



Torsion \implies **Contraintes tangentes**

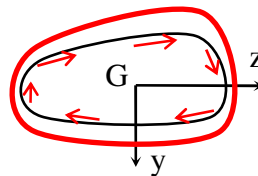
Profil mince fermé mono cellulaire

Flux de cisaillement constant : $\tau_B e_B = \tau_A e_A$



Première formule de BREDT

$$\tau = -\frac{T}{2Se} \quad \tau_{\max} = \frac{T}{2Se_{\min}}$$



Deuxième formule de BREDT

$$\frac{d\theta}{dx} = -\frac{T}{GJ} \quad J = \frac{4S^2}{\oint \frac{ds}{e}}$$

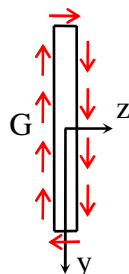
Anneau : $J = I_G$

Profil mince ouvert

$$J = \frac{he^3}{3}$$

$$\tau_{\max} = \frac{T}{J} e$$

Le long du bord extérieur vertical



$$\frac{d\theta}{dx} = -\frac{T}{GJ}$$

Profils composés :

$$J = \sum \frac{he^3}{3}$$